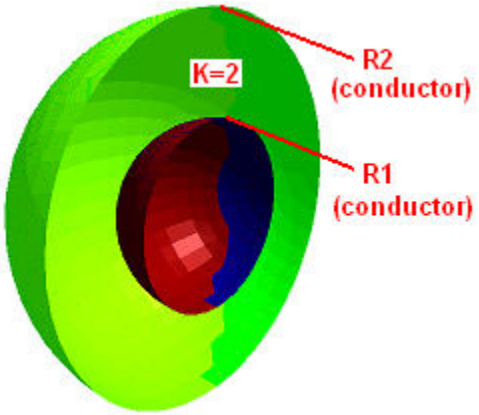
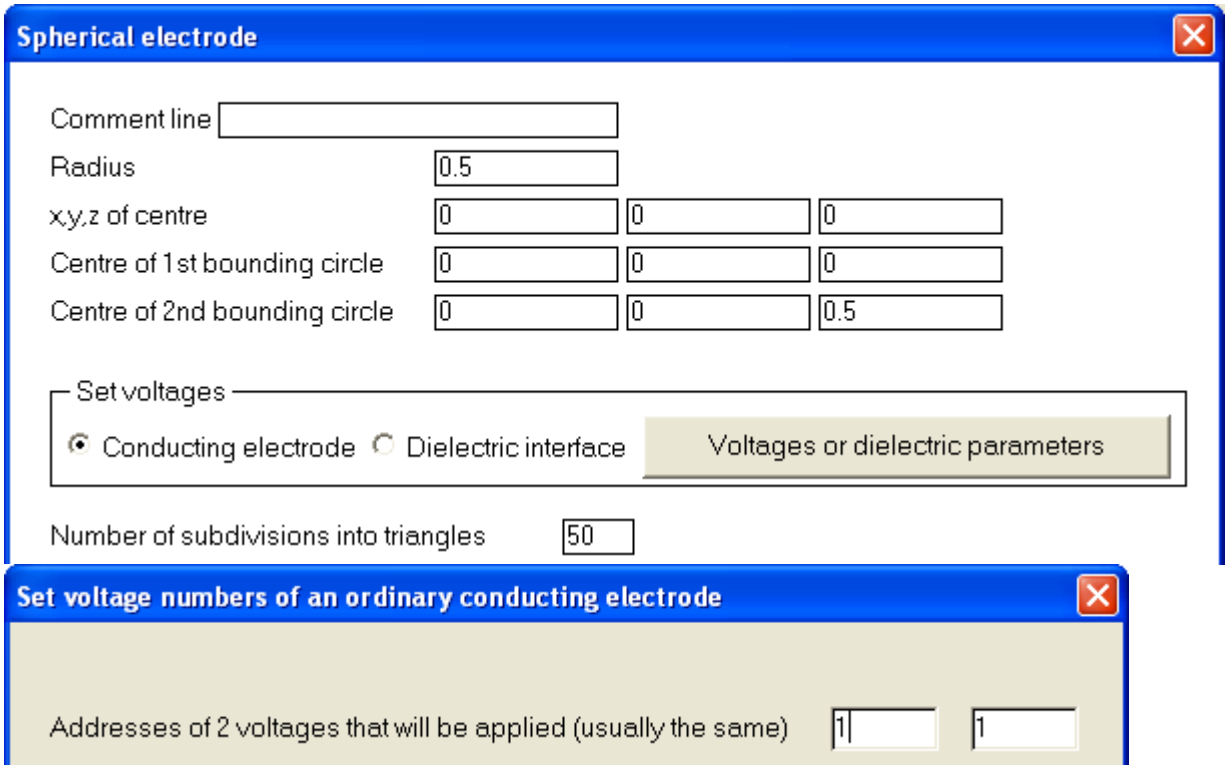
Simulating Dielectrics in CPO

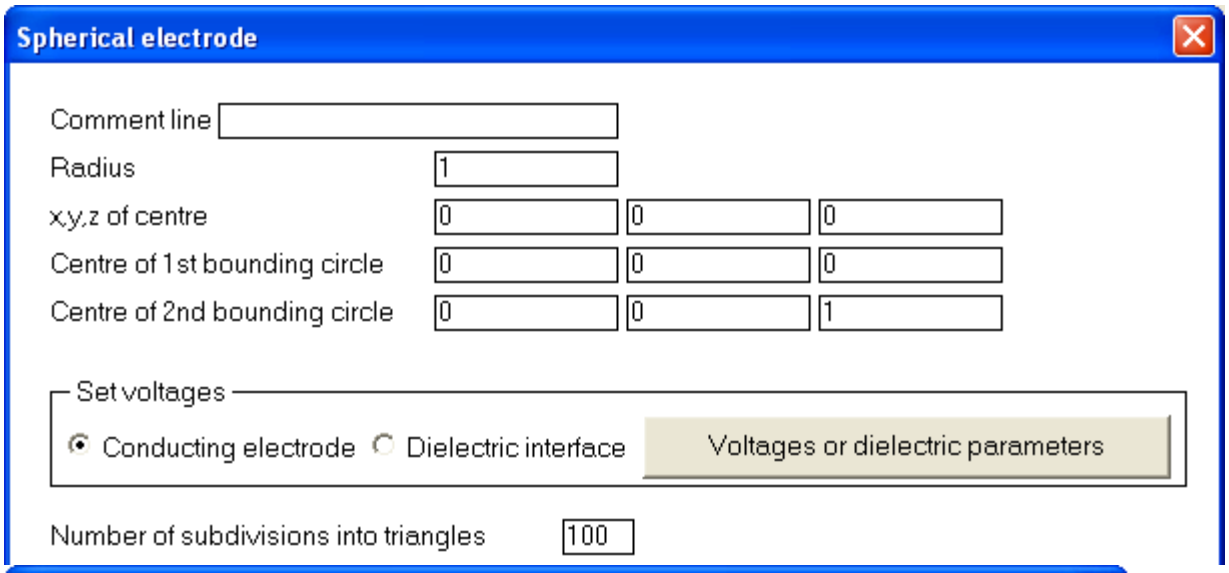
Spherical capacitor with dielectric in gap (22nd benchmark test for CPO3D)

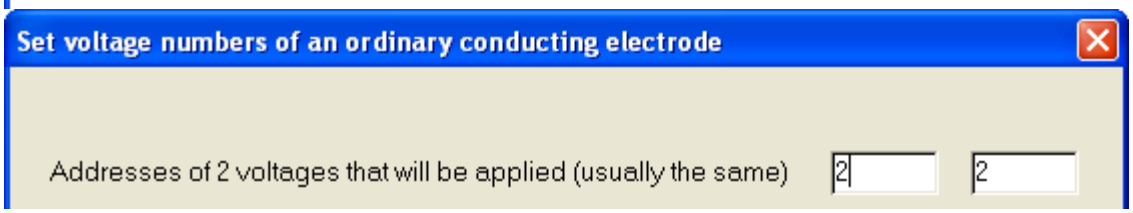
In this problem, inner and outer conducting spheres have radii of 0.5 and 1 mm respectively and voltages of 1 and 0 respectively. The dielectric between them has a dielectric constant of K = 2. A cutaway view of this system is shown below:



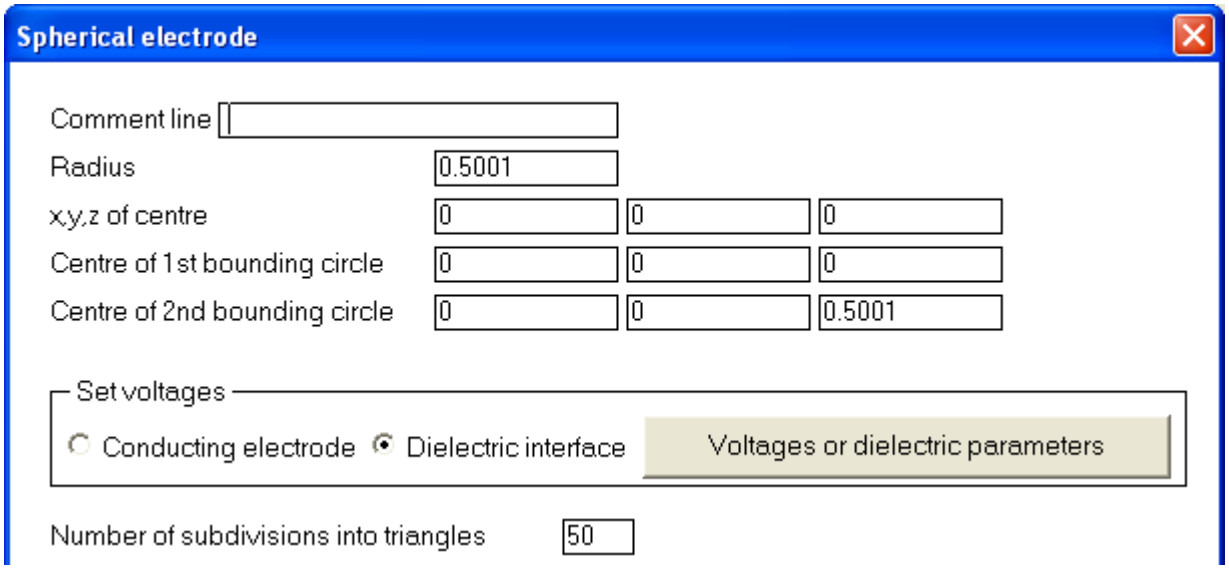
This is modeled with four shapes. The first two shapes are the inner and outer conducting spheres, respectively:

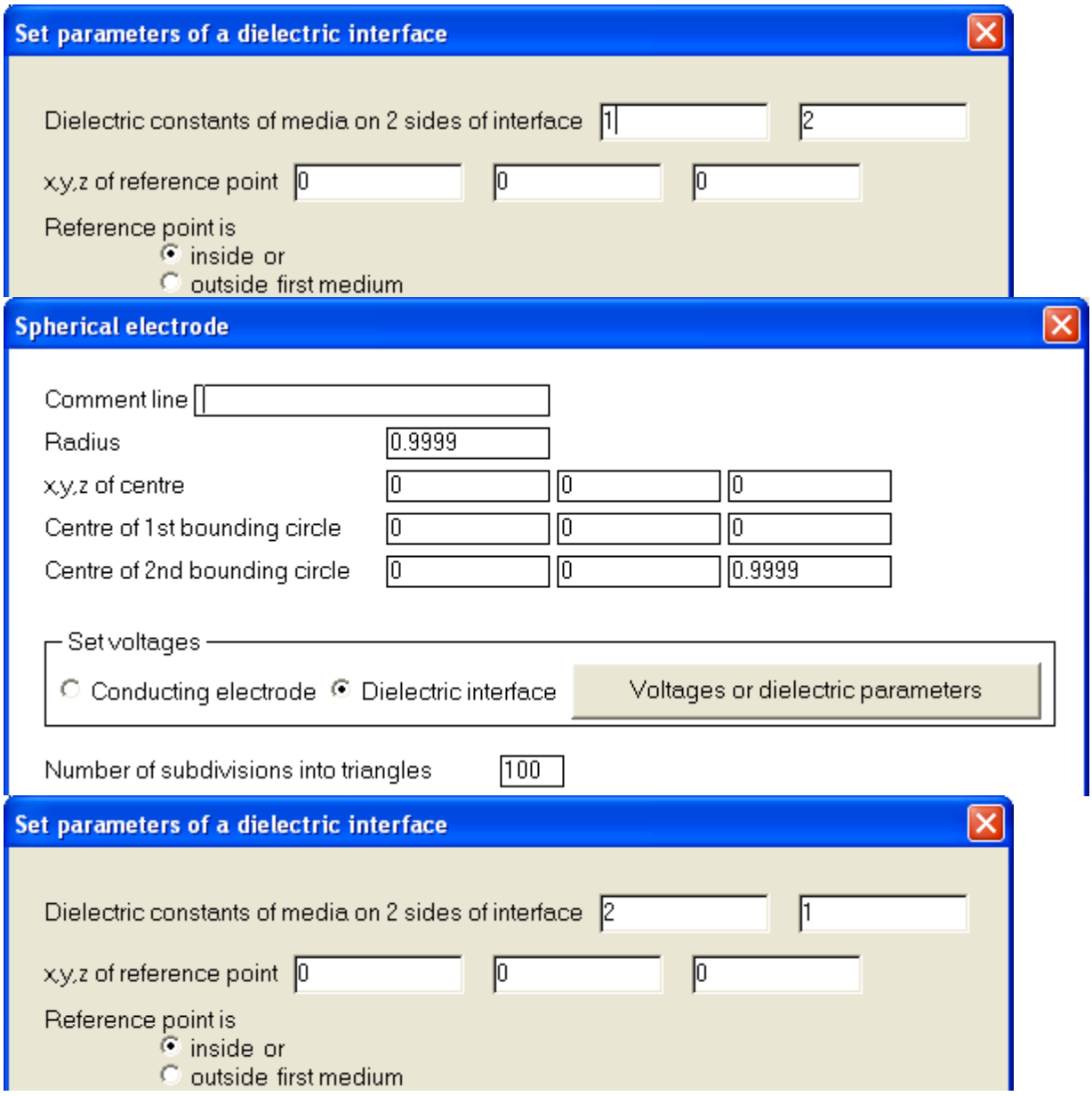






The last two shapes define the dielectric boundaries--that is, the surfaces that separate regions of materials with different dielectric constants. These surfaces are defined very close to (almost touching) the inner and outer radii: 0.5001 and 0.9999, which are almost at the minimum distance that the program will allow as discussed in the Help file.





The total number of segments is 308 in practice. The inaccuracy used for evaluation of the fields at the dielectric interfaces is 0.001 (but see below).

The theoretical capacitance of a sphere of radius 1 m is C = 4*pi*epsilon0 = 1.11265003E-10 farad = C0, say. If the dielectric were to completely fill the gap the capacitance of the present system would be K*1E-3*C0. It can be shown that for radii r1 and r2 of the conducting spheres and radii s1 and s2 of the dielectric interfaces, the capacitance is C0/((1/r1 - 1/s1) + (1/s1 - 1/s2)/K +(1/s2 - 1/r2)). In the present case this gives 1.99900*1E-3*C0 = 2.22419E-13 farad. 4 reflection planes are used here, so the charge given by the program is 1/16 of the full charge. Therefore for a voltage difference of 1V, the program charge should be 1.39012E-14 coulomb.

In fact the cumulative charge for the inner conducting sphere given by the program is 1.3872418E-14-- this is the cumulative charge for segment 50 in the output file, in the 3rd (last, be careful) listing of charges. The error is therefore **0.2%**. The same error is achieved for K = 10. When the radii of the dielectric interfaces are changed to 0.6 and 0.9 the error is 0.1%. But when the radii are changed and K

= 10, the errors are larger, see below.

Higher accuracy can of course be obtained by using more segments.

When the dielectric is removed the capacitance is given with an error of 0.005% and in a computing time of 1.6s (as opposed to the time of 15s for the dielectric calculation). This illustrates a general observation in the examples which we have tested and which involve curved electrodes: the dielectric calculations do not have the same very high accuracy of the normal non-dielectric calculations. Also, dielectric calculations with CPO3D are always much slower than the analogous non-dielectric calculations.

The inaccuracy used for evaluation of the fields at the dielectric interfaces can be selected by the User (at the bottom of the 'tracing accuracy' sheet). The default value is 0.001. Note that this inaccuracy is not the same as the inaccuracy used in calculating the surface charges of the conducting electrodes, which is always 1.E-7 and which cannot usually be changed by the User. Here are the results of using different 'dielectric interface' inaccuracies, still using only 308 segments:

inaccuracy	time (sec)	error, %
0.1	7.7	-30
0.01	8.5	-33
0.003	15	-0.18
0.001	15	-0.21
0.0001	41	-0.22
0.00001	50	-0.21

Same test, but K = 10, radii of dielectric = 0.6, 0.9:

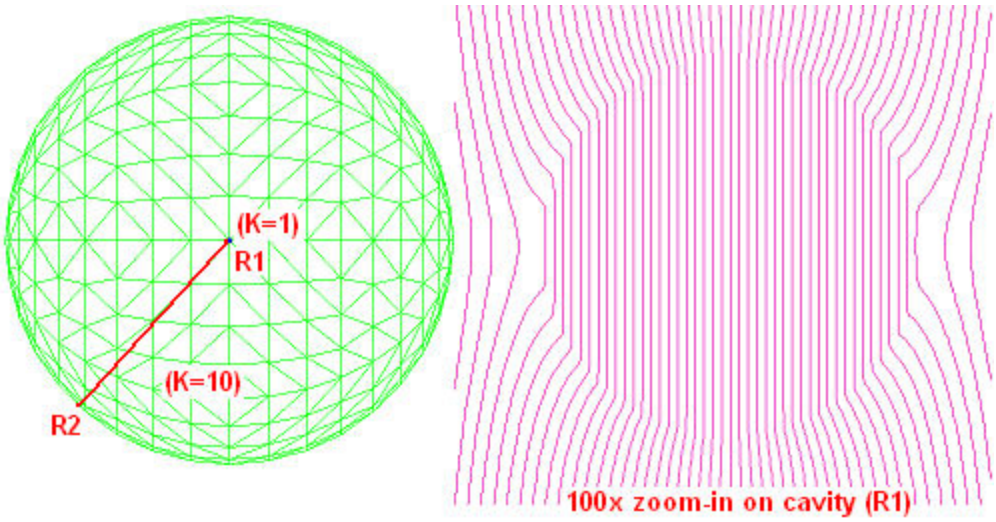
inaccuracy	time (sec)	error, %
0.1	8	-2.09
0.01	9	0.60
0.003	16	1.37
0.001	16	1.77
0.0001	43	1.63
0.00001	55	1.56

The default value in CPO3D for this inaccuracy is 0.001, but it might sometimes be better to change this to 0.0001 (at the bottom of the 'tracing accuracy' sheet).

The above is based in part on the documentation included in the TEST3D22.DAT example in CPO.

24th benchmark test for CPO3D, field in a cavity inside a dielectric.

A uniform field of 1V/mm is created by applying a linear field to a sphere of radius 5mm. The sphere is nearly filled with a medium of dielectric constant K = 10. The inaccuracy used for evaluation of the fields at the dielectric interfaces is 0.001 (but see below).



A cavity of radius 0.05mm exists at the centre of the dielectric cylinder.

As the text-books say, the field inside the cavity should be uniform, of strength E_cavity = E_ext*3*K/(2*K + 1) In the present example this gives E_cavity = 1.42857V/mm.

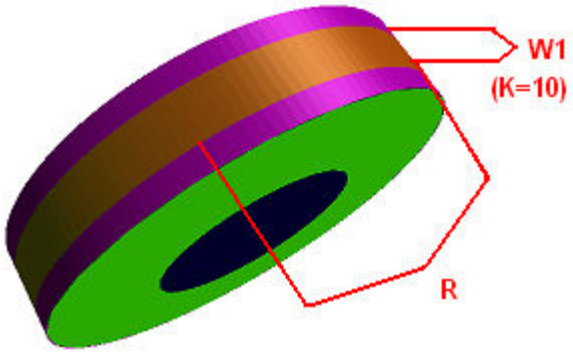
More exactly, the gaps at the end of the dielectric cylinder cause the internal field to be reduced to 0.9982 (= (d1 + d2)/(K*d1 + d2), where d1 and d2 are the lengths of the combined air gaps and the dielectric respectively). In the present simulation the internal field is slightly different, 0.996083 (found by removing the cavity). Therefore the expected internal field is 1.42298.

It can be seen from the output file that the calculated internal field is 1.42923 at the centre of the cavity, giving an error of 0.4%.

Higher accuracy can of course be obtained by using more segments.

The inaccuracy used for evaluation of the fields at the dielectric interfaces can be selected by the User (at the bottom of the 'tracing accuracy' sheet). The default value is 0.001, but it might sometimes be better to change this to 0.0001 (at the bottom of the 'tracing accuracy' sheet). Note that this inaccuracy is not the same as the inaccuracy used in calculating the surface charges of the conducting electrodes, which is always 1.E-7 and which cannot usually be changed by the User.

23rd benchmark test for CPO3D, parallel plate capacitor with dielectric in gap.



A circular disc of radius 0.5mm is at z = -0.1mm, at -0.5V. A similar disc is at z = +0.1mm, at +0.5V. The number of segments in each of these discs is 96. The inaccuracy used for evaluation of the fields at the dielectric interfaces is 0.001 (but see below).

A dielectric medium (K = 2) has been added between z = -0.9999 and +0.9999mm (the distances from the conducting surfaces are then almost at the minimum that the program will allow, see Help). The total number of segments is 608.

The textbook formula for the capacitance of this parallel plate capacitor is K*(A/d)*epsilon0, where K is the dielectric constant of the medium between the plates and A and d are the area and spacing. In the present case this gives 3.477031E-14 farad.

However in practice this is an under-estimate (see also the remarks at the end), because it ignores the extra charges on the inside surfaces caused by the fringe fields at the edges and it also ignores the charges on the outer surfaces (which are significant near the edges). Therefore in this benchmark test we have added surrounds of radius 1mm to the inner discs. We have also added electrodes at z = +/-0.2mm to create thick plates. In this test we shall only be concerned with the charges on the inner discs. In a subsidiary test the dielectric was removed and the cummulative charge on the inner disc was found to be 4.3352319E-15 -this is the cumulative charge for segment 96 in the output file, in the 3rd (last, be careful) listing of charges. This is 1/8 of the full charge because the x, y and x=y reflection planes have been used. It is 0.25% smaller than the texbook result 4.346288E-15. When the dielectric medium is included the effective value of K is (d1 + d2)/(d1 + d2/K), where d1 and d2 are the depths of the combined air gaps and the dielectric respectively. In the present case d1 = 0.0002, d2 = 0.1998, K = 2, so the effective constant is 1.998 and the expected charge is 8.683892E-15. The program gives 8.6611945E-15, which is too small by 0.26%. Higher accuracy can of course be obtained by using more segments.

inaccuracy	time (sec)	error, %
0.1	27	-33
0.01	29	-26
0.003	43	-0.27
0.001	43	-0.26
0.0001	130	-0.23
0.00001	168	-0.22